T.C.
HARRAN ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

YÜKSEK LİSANS TEZİ

EKSPONANSİYEL FONKSİYON METODU ÜZERİNE BİR ÇALIŞMA

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MATEMATİK ANABİLİM DALI

ŞANLIURFA
2025

İÇİNDEKİLER

ÖZET

Yüksek Lisans Tezi

EKSPONANSİYEL FONKSİYON METODU ÜZERİNE BİR ÇALIŞMA

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Yıl: 2025, Sayfa: ??

Farklı disiplinlerde modellenen bazı problemler kısmı kısmi diferansiyel denklemlerle temsil edilir. Literatürde çalışılan bu kısmi diferansiyel denklemlerin çoğu doğrusal olmayan denklemlerdir. Eğer bu doğrusal olmayan kısmi diferansiyel denklemlerin bir çözümü veya çözümleri varsa, bu çözüme veya çözümlere ulaşmak çok önemlidir. Bu kısmi diferansiyel denklemlerin yaklaşık veya kesin çözümlerini bulmak için literatürde çeşitli yarı analitik yöntemler bulunmaktadır. Bu yöntemlerden biri de exp fonksiyon yöntemidir. Exp fonksiyon yöntemi veya exp yöntemi CDGSK denklemi, KdV benzeri denklem ve JM doğrusal olmayan diferansiyel denklemlere uygulanarak istenen yaklaşık çözüm veya kesin çözümler elde edildi.Son olarak, elde edilen çözüm veya çözümlerin iki boyutlu, üç boyutlu ve kontur grafikleri verilmiştir.

ANAHTAR KELİMELER: Exp fonksiyon yöntemi, Diferansiyel denklemler, dalga çözüm
yaklaşık çözüm, tam çözüm

ABSTRACT

MSc Thesis

A STUDY ON THE EXPONENTIAL FUNCTION METHOD

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Year: 2025, Page: ??

Some problems modeled in different disciplines are represented by partial differential equations. Most of these partial differential equations studied in the literature are nonlinear equations. If there is a solution or solutions to these nonlinear partial differential equations, it is very important to reach this solution or solutions. There are various semi-analytical methods in the literature to find approximate or exact solutions of these partial differential equations. One of these methods is the exp function method. The exp function method or exp method is applied to CDGSK equation, KdV alike equation and JM nonlinear differential equations to obtain the desired solution or solutions. Finally, two-dimensional, three-dimensional and contour graphs of the obtained solution or solutions are given.

KEY WORDS: Exp function method, differential equations, wave solution, approximate solutions, exact solutions

TEŞEKKÜR

Tez konusunun seçimi, yürütülmesi ve yazımı konusundaki yardımları ve yakın ilgisinden dolayı tez danışmanım Sayın Prof. Dr. Tanfer TANRIVERDİ'ye, tez jürimde görev alan değerli akademisyen hocalarıma, bana yaşamım boyunca öğreten herkese ve her zaman desteklerini esirgemeyen ailemdeki herkese ayrı ayrı teşekkür ederim.

ŞEKİLLER DİZİNİ

1. GİRİŞ

Bilinen temel tanımlar ve formüller ifade edilecektir.

Tanım 1 "Bağımlı değişkenin bir veya daha fazla bağımsız parametreye göre türevlerini ihtiva eden denkleme türevli denklem denir. Denklem bir bağımsız parametreli ise denkleme sıradan (bayağı) diferansiyel denklem denir. Diğer halde, denkleme parçalı türevli denklem denir. Teknik olarak, $k = 1, 2, 3, \ldots, n$ için x_k verilsin ve $u = u(x_1, x_2, \ldots, x_n)$ ise

$$F(x_1, x_2, \ldots, x_n, u, u_{x_1}, \ldots, u_{x_n}, \ldots, \frac{\partial^n u}{\partial x^n}) = 0$$

denklemi n. meretebeden parçalı türevli denklem denir". (Biz, 2019)

$$\frac{\partial u}{\partial x} = h(x),\ u_t = cu_{xy}$$

veya

$$xu_t + t^2u^2 = \sin(x)$$

ifadeleri parçalı diferansiyel denklemlere birer örnek olarak verilebilir. İkinci mertebeden parçalı türevli denklemler oldukça önemlidir. Aşağıdaki gibi sınıflandırılabilir.

Tanım 2 "İkinci mertebeden

$$D(x,y)\frac{\partial^2 u}{\partial x^2} + E(x,y)\frac{\partial^2 u}{\partial x \partial y} + F(x,y)\frac{\partial^2 u}{\partial y^2} + f(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}) = 0$$

parçalı türevli denklemde

- $E^2 - 4DF < 0$ ise eliptik,
- $E^2 - 4DF = 0$ ise parabolik,
- $E^2 - 4DF > 0$ ise hiperboliktir". (Biz, 2019)

Tanım 3 "Tek parametreli $h(x)$ fonksiyonun $x = x_0$ komşuluğundaki Taylor seri açılımı

$$h(x) = h(x_0) + h'(x_0)(x - x_0) + \frac{h''(x_0)}{2!}(x - x_0)^2 + \frac{h'''(x_0)}{3!}(x - x_0)^3 + \cdots$$
$$+ \frac{h^n(x_0)}{n!}(x - x_0)^n + \cdots .$$

Eğer $x_0 = 0$ olarak alınırsa seri literatürde Maclaurin seri açılımı olarak bilinir”. (Biz, 2019)

Tanım 4 “İki parametreli $h(x, y)$ fonksiyonunun $(x, y) = (x_0, y_0)$ noktasında Taylor serisi

$$\begin{aligned} h(x, y) = {} & h(x_0, y_0) + h_x(x_0, y_0)(x - x_0) + h_y(x_0, y_0)(y - y_0) \\ & + \frac{h_{xx}(x_0, y_0)(x - x_0)^2 + h_{xy}(x_0, y_0)(x - x_0)(y - y_0) + h_{yy}(x_0, y_0)(y - y_0)^2}{2!} \\ & + \cdots \end{aligned}$$

biçimindedir”. (Biz, 2019)

2. ÖNCEKİ ÇALIŞMALAR

Doğadaki problemlerin çoğu türevli denklemlerle ifade edilebilir. Ele alınan problemlerin bir çözüme sahip olma şartı oldukça önemlidir (Davis, 1962; Whitham, 1974; Logan, 1994; Debnath, 2005, stabilite analizi için (Teschl, 2024), popülasyon için (Waltman, 1983) ve kesirli analiz için (Podlubny, 1998) kaynaklarına bakılabilir.

Lineer olmayan dalga denklemlerin dalga çözümlerini elde etmek için exp fonksiyon metodu veya kısaca exp metot ilk olarak (He ve Wu, 2006) yılında ele alınmıştır.

İlk olarak, Coudrey-Dodd-Gibbon-Sawada-Kotera denklemi

$$A_t + A_{xxxxx} + 30AA_{xxx} + 30A_xA_{xx} + 180A^2A_x = 0$$

denklemi ele alınacaktır. CDGSK denkleminin fiziksel yorumu için (Aiyer vd.,1986) çalışmasına bakılabilir. CDGSK denklemi farklı yöntemlerle ele alınmıştır. Bu yöntemlerden bazıları CDGSK denkleminin tanh metodu (Wazwaz, 2006), Hirota metodu (Jiang ve Bi, 2010; Salas vd., 2011; Ma vd., 2016), Hirota metodu ve Hereman metodu (Wazwaz, 2008), Lie simetri analiz metodu (Baleanu, vd., 2018), Bernoulli alt denklem fonksiyon metodu (Baskonus vd., 2022), Kudryashov metodu (Yıldız, 2019), exp metot (Xu,, 2008; Salas, 2008a), projektif Riccati denklem metodu (Salas, 2008b), (G'/G) açılım metodu (Naher, 2011; Shakeel ve Mohyud-Dın, 2015), He varyasyonel iterasyon metodu (Safari, 2011), $\exp(-\phi(\xi))$ açılım metodu (Hedli ve Kadem, 2020), Homojen denklem metodu (Zayed ve Alurrfi, 2014) çalışmalarında ele alınmıştır.

İkinci problem olarak,

$$U_{xxx} + \alpha_2U_{yyy} + \alpha_3UU_x + \alpha_4U_t = 0$$

Korteweg-de Vries (KdV) benzeri iki boyutlu kısmi türevli denklem ele alınacaktır. Bir boyutlu KdV denkleminin exp fonksiyon metodu (Salas, 2009; Kutluay ve Esen, 2009; Sajid vd., 2022), klasik yöntem (Miura, 1976; Shi, 2024), Bäcklund transform (Brauer, 2000), Painleve analysis (Salas vd., 2013), sine-Gordon expansion method (Bulut vd., 2016) ve genelleştirilmiş exp method (Marinakis, 2008) çalışmalarına bakılabilir.

Üçüncü problem olarak,

$$V_{xxxy} + A_2 V_y V_{xx} + A_3 V_x V_{xy} + A_4 V_{yt} - A_5 V_{xz} = 0 \qquad (2.1)$$

plazma fizikte önemli bir yer tutan (3+1) boyutlu Jimbo-Miwa (JM) denklemi ele alınacaktır (Jimbo ve Miwa, 1983). JM denklemi değişik yöntemlerle çalışılmıştır. Örneğin, Painlevé method (Xu,2006), genişletilmiş rasyonel açılım metodu (Wang vd., 2007), exp fonksiyon metodu (Öziş ve Aslan, 2008), Riccati denklem dönüşüm metodu (Li ve Dai, 2010), (G'/G) açılım metodu (Abazari, 2012; Khalique ve Moleleki 2020; Naher vd., 2014), Hirota bilineer metodu (Wazwaz, 2008; Ma,1016; Yue vd., 2019; Xu vd., 2023), lineer süperpozisyon ilkesi ve Hirota metodu (Kuo ve Ghanbari, 2019), Kudryashov metodu (Ali vd., 2018), (1/G')-expansion metodu (Yokuş ve Durur 2020), singüler manifold ve grup dönüşüm metotları (Rashed vd., 2022), çoklu exp fonksiyon metodu (Mukam vd., 2022), değiştirilmiş direkt metod (Ma, 2005) çalışmalarına bakılabilir. Kesirli türeve sahip denklemlerin Laplace yöntemi ile (Tanriverdi vd., 2021; Güneş, 2021; İşleyen, 2024), yarı analitik yöntemler Chakraverty vd., 2019), basit denklem metodu (Hoşer, 2023), rezidü yöntemiyle çözüm (Tanriverdi, 2001; Tanriverdi ve Mcleod, 2007; Tanriverdi ve Mcleod, 2008; Tanriverdi, 2009; Tanriverdi, 2019; Tanriverdi, 2019a), klasik yöntemle analiz (Tanriverdi, 2012; Tanriverdi, 2012a; Tanriverdi, 2017; Tanriverdi, 2019), Laplace dönüşüm metodu (Tanriverdi, 2018; Tanriverdi, 2018a), serilerin asimtotik analiz, (Merca ve Tanriverdi, 2013), topolojik atış metodu (Hastings ve McLeod, 2011; Tanrıverdi ve Mcleod, 2010), dönüşüm metodu (Tanriverdi ve Ağırağaç, 2018; Biz, 2019), iterasyon tekniği (Alıcı ve Tanriverdi, 2020; Alıcı ve Tanriverdi, 2021), asimtotik analiz tekniği (Tanriverdi, 2021), Riemann zeta hiptotezi (Tanriverdi, 2021), Değiştirilmiş eksponansiyel fonksiyon metodu (Muhamad vd., 2023), Bernoulli alt-denklem metodu (Başkonuş vd., 2022; Başkonuş vd., 2022a; Mahmud vd., 2023), genişletilmiş rasyonel sinh-cosh ve değiştirilmiş ve genişletilmiş tanh-function metodu (Mahmud vd., 2023; Mahmud vd., 2023a) çalışmalarına bakınız.

3. GEREÇ VE YÖNTEM

Burada, diferansiyel denklemlerin dalga çözümlerininin yaklaşığını veya tam çözümünü bulmak için Exp fonksiyon metodu anlatılacaktır.

3.1. Eksponansiyel Fonksiyon Metodu

Bu kısımda, Eksponansiyel Fonksiyon Metodu (EFM) anlatılacaktır. EFM lineer olmayan kısmi diferansiyel denklemlerin periyodik ve soliton çözümlerini elde etmek için ilk olarak (He ve Wu, 2006) çalışmasıyla literatüre kazandırılmıştır.

$$R(U, U_x, U_{xx}, U_{xt}, U_{tt}, \ldots) = 0 \tag{3.1}$$

denklemini ele alalım. α ve β sabit olmak üzere (**??**) denklemine

$$U(x,t) = U(\xi),\ \xi = \alpha x + \beta t \tag{3.2}$$

dönüşümü uygulanırsa

$$H\big(U, U', U'', U''', \ldots\big) \tag{3.3}$$

bayağı diferansiyel denklemi elde edilir. (**??**) denkleminin çözümü

$$U(\xi) = \frac{\sum_{n=-c}^{d} a_n e^{n\xi}}{\sum_{n=-p}^{q} b_n e^{n\xi}} \tag{3.4}$$

şeklinde olsun. Burada, c, d, p ve q parametreleri dengeleme ile belirlenmesi gereken pozitif tamsayılar ve a_n ve b_n ise bilinmeyen sabitlerdir. (**??**) denklemindeki en düşük dereceli nolineer terim ile mertebesi en düşük olan lineer terimin eksponansiyel fonksiyonlarının dengelenmesi ile c ve p parametrileri belirlenir. Benzer olarak, (**??**) denklemindeki en yüksek dereceli nolineer terim ile mertebesi en yüksek olan lineer terimin eksponansiyel fonksiyonlarının dengelenmesi ile d ve q parametrileri belirlenir.

Dolayısıyla, (**??**) denklemi (**??**) denkleminde yerine yazılır. Gerekli işlemler yapıldıktan sonra eksponansiyel terimli ifadelerin katsayıları sıfıra eşitlenerek a_n, b_n, α ve β bilinmeyen sabitleri belirlenir. Bu katsayılar belirlendikten sonra istenilen (**??**) çözümü elde edilir.

3.2. Denge Prensibinin Uygulamaları

Bu parametler ile ilgili uygulamalar aşağıda örneklerle açıklanmıştır.

Örnek 1 KdV denklemi

$$U_t + 6UU_x + U_{xxx} = 0 \tag{3.5}$$

denklemine

$$z(x,t) = z(x - \beta t) = z(\xi) \tag{3.6}$$

dönüşümü tatbik edilirse

$$-z\beta U' + 6zUU' + z^3U''' = 0 \tag{3.7}$$

bayağı diferansiyel denklemi elde edilir. Ayrıca, (**??**) denklemi integre edilir ve integral sabiti sıfır alınırsa

$$-2z\beta U + 3zU^2 + 2z^3U'' = 0 \tag{3.8}$$

bulunur. Dengeleme prensibi gereğince

$$\begin{aligned} U^2 &= \frac{a_{-c}^2 e^{-2c\xi} + \cdots}{b_{-p}^2 e^{-2p\xi} + \cdots}, \\ U'' &= \frac{Ge^{-(c+3p)\xi} + \cdots}{Ee^{-4p\xi} + \cdots} \end{aligned} \tag{3.9}$$

denklemlerinden

$$2c + 2p = c + 3p, \ p = c \tag{3.10}$$

bulunur ve

$$\begin{aligned} U^2 &= \frac{\cdots + a_d^2 e^{2d\xi}}{\cdots + b_q^2 e^{2q\xi}}, \\ U'' &= \frac{\cdots + He^{(d+3q)\xi}}{\cdots + Fe^{4q\xi}} \end{aligned} \tag{3.11}$$

denklemlerinden

$$2d + 2q = d + 3q,\ d = q \tag{3.12}$$

elde edilir. Böylece, $p = c = d = q = 1$ alınabilir.

Örnek 2 Benjamin-Ono denklemi

$$U_t + HU_{xx} + UU_x = 0 \tag{3.13}$$

denklemine

$$\begin{aligned} &U(x,t) = U(\eta), \\ &\quad \eta = kx - ct \end{aligned} \tag{3.14}$$

dönüşümü tatbik edilirse

$$-cU' + Hk^2U'' + kUU' = 0 \tag{3.15}$$

bayağı diferansiyel denklemi elde edilir. Ayrıca, (**??**) denklemi integre edilir ve integral sabiti sıfır alınırsa

$$-2cU + 2Hk^2U' + kU^2 = 0 \tag{3.16}$$

bulunur. Dengeleme gereğince

$$\begin{aligned} U^2 &= \frac{a_{-c}^2 e^{-2c\eta} + \cdots}{b_{-p}^2 e^{-2p\eta} + \cdots}, \\ U' &= \frac{Ae^{-(c+p)\eta} + \cdots}{De^{-2p\eta} \cdots +} \end{aligned} \tag{3.17}$$

denklemlerinden

$$c + p = 2c,\ p = c \tag{3.18}$$

bulunur ve

$$\begin{aligned} U^2 &= \frac{\cdots + a_d^2 e^{2d\eta}}{\cdots + b_q^2 e^{2q\xi}}, \\ U' &= \frac{\cdots + Be^{(d+q)\eta}}{\cdots + Ce^{2q\eta}} \end{aligned} \tag{3.19}$$

denklemlerinden

$$d + q = 2d, \ d = q \tag{3.20}$$

elde edilir. $p = c = d = q = 1$ değerini alabilir.

Örnek 3 Chafee-Intante denklemi

$$U_t - U_{xx} + \lambda(U^3 - U) = 0 \tag{3.21}$$

denklemine

$$\xi = kx - ct \tag{3.22}$$

dönüşümü tatbik edilirse

$$-cU' - k^2U'' + \lambda(U^3 - U^2) = 0 \tag{3.23}$$

bayağı diferansiyel denklemi elde edilir. Ayrıca, (**??**) denklemi integre edilir ve integral sabiti sıfır alınırsa

$$-4cU - 4k^2U' + \lambda(U^4 - 2U^3) = 0 \tag{3.24}$$

bulunur. Dengeleme prensibi gereğince

$$\begin{aligned} U^4 &= \frac{a_{-c}^4 e^{-4c\xi} + \cdots}{b_{-p}^4 e^{-4p\xi} + \cdots}, \\ U' &= \frac{Ae^{-(c+p)\xi} + \cdots}{De^{-2p\xi} \cdots +} \end{aligned} \tag{3.25}$$

denklemlerinden

$$c + 3p = 4c, \ p = c \tag{3.26}$$

bulunur ve

$$\begin{aligned} U^4 &= \frac{\cdots + a_d^4 e^{4d\xi}}{\cdots + b_q^4 e^{4q\xi}}, \\ U' &= \frac{\cdots + Be^{(d+q)\xi}}{\cdots + Ce^{2q\xi}} \end{aligned} \tag{3.27}$$

denklemlerinden

$$d + 3q = 4d, \; d = q \tag{3.28}$$

elde edilir. $p = c = d = q = 1$ olarak alınabilir.

4. BULGULAR

Burada, EFM metotları lineer olmayan diferansiyel denklemlere uygulanarak yaklaşık ve tam çözümler elde edilecektir.

4.1. CDGSK Denklemi

$$A_t + A_{xxxxx} + 30AA_{xxx} + 30A_xA_{xx} + 180A^2A_x = 0 \tag{4.1}$$

denklemine

$$\xi = \alpha x - ct,\ A(x,t) = A(\xi) \tag{4.2}$$

dönüşümü tatbik edilirse

$$\alpha^5A^{(5)} + 30\alpha^3AA''' + 30\alpha^3A'A'' + 180\alpha A^2A' - cA' = 0 \tag{4.3}$$

bayağı diferansiyel denklemi elde edilir.

(**??**) denklemine dengeleme perensibi uygulanırsa

$$\begin{aligned} A^2 &= \frac{a_{-c}^2e^{-2c\xi}+\cdots}{b_{-p}^2e^{-2p\xi}+\cdots}, \\ A^{(5)} &= \frac{Me^{-(c+31p)\xi}+\cdots}{Pe^{-32p\xi}+\cdots} \end{aligned} \tag{4.4}$$

denklemlerinden

$$2c + 30p = c + 31p,\ p = c \tag{4.5}$$

bulunur ve

$$\begin{aligned} A^2 &= \frac{\cdots+a_d^2e^{2d\xi}}{\cdots+b_q^2e^{2q\xi}}, \\ A^{(5)} &= \frac{\cdots+Ne^{(d+31q)\xi}}{\cdots+Re^{32q\xi}} \end{aligned} \tag{4.6}$$

denklemlerinden

$$2d + 30q = d + 31q, d = q \tag{4.7}$$

elde edilir. Şimdi, $p = c = d = q = 1$ alalım. Bu durumda,

$$A(\xi) = \frac{a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi}} \tag{4.8}$$

elde edilir. Burada amacımız,

$$a_{-1},\ a_0,\ a_1,\ b_{-1},\ b_0,\ b_1 \tag{4.9}$$

katsayılarını bulmaktır. Ayrıca,

$$A^2 \quad = \frac{a_{-c}^2 e^{-2c\xi} + \cdots + a_d^2 e^{2d\xi}}{b_{-p}^2 e^{-2p\xi} + \cdots + b_q^2 e^{2q\xi}}, \tag{4.10}$$

$$\begin{aligned} A'(\xi) = & \frac{-(a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi})(-b_{-1}e^{-\xi} + b_1 e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi})^2} \\ & + \frac{-a_{-1}e^{-\xi} + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi}}, \end{aligned} \tag{4.11}$$

$$\begin{aligned} A''(\xi) = & \frac{2(a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi})(-b_{-1}e^{-\xi} + b_1 e^{\xi})^2}{(b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi})^3} \\ & - \frac{2(-a_{-1}e^{-\xi} + a_1 e^{\xi})(-b_{-1}e^{-\xi} + b_1 e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi})^2} \\ & - \frac{(a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi})(b_{-1}e^{-\xi} + b_1 e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi})^2} \\ & + \frac{a_{-1}e^{-\xi} + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi}}, \end{aligned} \tag{4.12}$$

$$\begin{aligned}
A'''(\xi) =& \frac{-6(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^3}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^4} \\
&+ \frac{6(-a_{-1}e^{-\xi}+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^2}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&+ \frac{6(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&- \frac{3(a_{-1}e^{-\xi}+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&- \frac{(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&- \frac{3(-a_{-1}e^{-\xi}+a_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&+ \frac{-a_{-1}e^{-\xi}+a_1e^{\xi}}{b_{-1}e^{-\xi}+b_0+b_1e^{\xi}},
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
A^{(4)}(\xi) =& \frac{24(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^4}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^5} \\
&- \frac{24(-a_{-1}e^{-\xi}+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^3}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^4} \\
&+ \frac{-36(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^2(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^4} \\
&+ \frac{12(a_{-1}e^{-\xi}+a_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})^2}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&+ \frac{8(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})^2}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&+ \frac{24(-a_{-1}e^{-\xi}+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&+ \frac{6(a_{-1}a_0e^{-\xi}+a_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})^2}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^3} \\
&- \frac{8(-a_{-1}e^{-\xi}+a_1e^{\xi})(-b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&- \frac{6(a_{-1}e^{-\xi}+a_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&- \frac{(a_{-1}e^{-\xi}+a_0+a_1e^{\xi})(b_{-1}e^{-\xi}+b_1e^{\xi})}{(b_{-1}e^{-\xi}+b_0+b_1e^{\xi})^2} \\
&+ \frac{a_{-1}e^{-\xi}+a_1e^{\xi}}{b_{-1}e^{-\xi}+b_0+b_1e^{\xi}},
\end{aligned} \tag{4.14}$$

$$
\begin{aligned}
A^{(5)}(\xi) = & \frac{-120(a_{-1}e^{-\xi} + a_0 + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^5}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^6} \\
& + \frac{120(-a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^4}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^5} \\
& + \frac{240(a_{-1}e^{-\xi} + a_0 + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^3(b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^5} \\
& - \frac{60(a_{-1}e^{-\xi} + a_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})^3}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^4} \\
& - \frac{60(a_{-1}e^{-\xi} + a_0 + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^3}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^4} \\
& - \frac{180(-a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^2(b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^4} \\
& - \frac{90(a_{-1}e^{-\xi} + a_0 + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})^2}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^4} \\
& + \frac{60(-a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})^2}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^3} \\
& + \frac{60(a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^3} \\
& + \frac{30(a_{-1}e^{-\xi} + a_0 + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^3} \\
& + \frac{30(-a_{-1}e^{-\xi} + a_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})^2}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^3} \\
& - \frac{15(a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^2} \\
& - \frac{(a_{-1}e^{-\xi} + a_1e^{\xi})(-b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^2} \\
& - \frac{15(-a_{-1}e^{-\xi} + a_1e^{\xi})(b_{-1}e^{-\xi} + b_1e^{\xi})}{(b_{-1}e^{-\xi} + b_0 + b_1e^{\xi})^2} \\
& + \frac{-a_{-1}e^{-\xi} + a_1e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1e^{\xi}}
\end{aligned} \tag{4.15}
$$

(**??**)-(**??**) denklemleri (**??**) denkleminde yerine yazılır ve $n = 1, \ldots, 11$ olmak üzere eksponansiyel ifadesinin katsayıları dikkate alınırsa aşağıdaki sistem elde edilir.

$$
\begin{aligned}
& 180\alpha a_{-1}^2 a_0 b_{-1}^3 + 30\alpha^3 a_{-1} a_0 b_{-1}^4 - c a_0 b_{-1}^5 + \alpha^5 a_0 b_{-1}^5 \\
& - 180\alpha a_{-1}^3 b_{-1}^2 b_0 - 30\alpha^3 a_{-1}^2 b_{-1}^3 b_0 + c a_{-1} b_{-1}^4 b_0 - \alpha^5 a_{-1} b_{-1}^4 b_0 = 0,
\end{aligned} \tag{4.16}
$$

$$
\begin{aligned}
&360\alpha a_{-1}a_0^2b_{-1}^3 + 360\alpha a_{-1}^2a_1b_{-1}^3 + 60\alpha^3a_0^2b_{-1}^4 + 240\alpha^3a_{-1}a_1b_{-1}^4\\
&- 2ca_1b_{-1}^5 + 32\alpha^5a_1b_{-1}^5 - 180\alpha^3a_{-1}a_0b_{-1}^3b_0 - 4ca_0b_{-1}^4b_0 - 26\alpha^5a_0b_{-1}^4b_0\\
&- 360\alpha a_{-1}^3b_{-1}b_0^2 + 120\alpha^3a_{-1}^2b_{-1}^2b_0^2 + 4ca_{-1}b_{-1}^3b_0^2 + 26\alpha^5a_{-1}b_{-1}^3b_0^2\\
&- 360\alpha a_{-1}^3b_{-1}^2b_1 - 240\alpha^3a_{-1}^2b_{-1}^3b_1 + 2ca_{-1}b_{-1}^4b_1 - 32\alpha^5a_{-1}b_{-1}^4b_0 = 0,
\end{aligned}
\tag{4.17}
$$

$$
\begin{aligned}
&180\alpha a_0^3b_{-1}^3 + 1080\alpha a_{-1}a_0a_1b_{-1}^3 + 450\alpha^3a_0a_1b_{-1}^4\\
&+ 540\alpha a_{-1}a_0^2b_{-1}^2b_0 + 540\alpha a_{-1}^2a_1b_{-1}^2b_0 - 90\alpha^3a_0^2b_{-1}^3b_0\\
&+ 180\alpha^3a_{-1}a_1b_{-1}^3b_0 - 9ca_1b_{-1}^4b_0 - 51\alpha^5a_1b_{-1}^4b_0 - 540\alpha a_{-1}^2a_0b_{-1}b_0^2\\
&- 6ca_0b_{-1}^3b_0^2 + 66\alpha^5a_0b_{-1}^3b_0^2 - 180\alpha a_{-1}^3b_0^3 + 90\alpha^3a_{-1}^2b_{-1}b_0^3\\
&+ 6ca_{-1}b_{-1}^2b_0^3 - 66\alpha^5a_{-1}b_{-1}^2b_0^3 - 540\alpha a_{-1}^2a_0b_{-1}^2b_1\\
&- 1080\alpha^3a_{-1}a_0b{-1}^3b_1 - 3ca_0b_{-1}^4b_1 - 237\alpha^5a_0b_{-1}^4b_1\\
&- 1080\alpha a_{-1}^3b_{-1}b_0b_1 + 450\alpha^3a_{-1}^2b_{-1}^2b_0b_1 + 12ca_{-1}b_{-1}^3b_0b_1\\
&+ 288\alpha^5a_{-1}b_{-1}^3b_0b_1 = 0,
\end{aligned}
\tag{4.18}
$$

$$
\begin{aligned}
&720\alpha a_0^2a_1b_{-1}^3 + 720\alpha a_{-1}a_0a_1^2b_{-1}^3 + 480\alpha^3a_1^2b_{-1}^4\\
&+ 360\alpha a_0^3b_{-1}^2b_0 + 2160\alpha a_{-1}a_0a_1b_{-1}^2b_0\\
&+ 540\alpha^3a_0a_1b_{-1}^3b_0 - 120\alpha^3a_0^2a_1b_{-1}^2b_0^2 + 120\alpha^3a_{-1}a_1b_{-1}^2b_0^2\\
&- 16ca_1b_{-1}^3b_0^2 + 46\alpha^5a_1b_{-1}^3b_0^2 - 360\alpha a_{-1}^2a_0b_0^3 + 180\alpha^3a_{-1}a_0b_{-1}b_0^3\\
&- 4ca_0b_{-1}^2b_0^3 - 26\alpha^5a_0b_{-1}^2b_0^3 - 60\alpha^3a_{-1}^2b_0^4 + 4ca_{-1}b_{-1}b_0^4\\
&+ 26\alpha^5a_{-1}b_{-1}b_0^4 - 840\alpha^3a_0^2b_{-1}^3b_1 - 1440\alpha^3a_{-1}a_1b_{-1}^3b_1\\
&- 8ca_1b_{-1}^4b_1 - 832\alpha^5a_1b_{-1}^4b_0 - 2160\alpha a_{-1}^2a_0b_{-1}b_0b_1\\
&- 420\alpha^3a_{-1}a_0b_{-1}^2b_0b_1 - 8ca_0b_{-1}^3b_0b_1 + 428\alpha^5a_0b_{-1}^3b_0b_1\\
&- 720\alpha a_{-1}^3b_0^2b_1 + 600\alpha^3a_{-1}^2b_{-1}b_0^2b_1 + 24ca_{-1}b_{-1}^2b_0^2b_1\\
&- 474\alpha^5a_{-1}b_{-1}^2b_0^2b_1 - 720\alpha a_{-1}^3b_{-1}b_1^2 + 960\alpha^3a_{-1}^2b_{-1}^2b_1^2\\
&+ 8ca_{-1}b_{-1}^3b_1^2 + 832\alpha^5a_{-1}b_{-1}^3b_1^2 = 0,
\end{aligned}
\tag{4.19}
$$

$$\begin{aligned}
&900\alpha a_0 a_1^2 b_{-1}^3 + 1620\alpha a_0^2 a_1 b_{-1}^2 b_0 + 1620\alpha a_{-1} a_1^2 b_{-1}^2 b_0 \\
&+ 930\alpha^3 a_1^2 b_{-1}^3 b_0 + 180\alpha a_0^3 b_{-1} b_0^2 + 1080\alpha a_{-1} a_0 a_1 b_{-1} b_0^2 \\
&+ 240\alpha^3 a_0 a_1 b_{-1}^2 b_0^2 - 180\alpha a_{-1} a_0^2 b_0^3 - 180\alpha a_{-1}^2 a_1 b_0^3 \\
&+ 30\alpha^3 a_0^2 b_{-1} b_0^3 + 180\alpha^3 a_{-1} a_1 b_{-1} b_0^3 - 14 c a_1 b_{-1}^2 b_0^3 + 14\alpha^5 a_1 b_{-1}^2 b_0^3 \\
&- 30\alpha^3 a_{-1} a_0 b_0^4 - c a_0 b_{-1} b_0^4 + \alpha^5 a_0 b_{-1} b_0^4 + c a_{-1} b_0^5 - \alpha^5 a_{-1} b_0^5 \\
&+ 180\alpha a_0^3 b_{-1}^2 b_1 + 1080\alpha a_{-1} a_0 a_1 b_{-1}^2 b_1 - 1800\alpha^3 a_0 a_1 b_{-1}^3 b_1 \\
&- 1080\alpha a_{-1} a_0^2 b_{-1} b_0 b_1 - 1080\alpha a_{-1}^2 a_1 b_{-1} b_0 b_1 - 1050\alpha^3 a_0^2 b_{-1}^2 b_0 b_1 \\
&- 1740\alpha^3 a_{-1} a_1 b_{-1}^2 b_0 b_1 - 28 c a_1 b_{-1}^3 b_0 b_1 - 392\alpha^5 a_1 b_{-1}^3 b_0 b_1 \\
&- 1620\alpha a_{-1}^2 a_0 b_0^2 b_1 + 960\alpha^3 a_{-1} a_0 b_{-1} b_0^2 b_1 - 6 c a_0 b_{-1}^2 b_0^2 b_1 \\
&- 174\alpha^5 a_0 b_{-1}^2 b_0^2 b_1 - 330\alpha^3 a_{-1}^2 b_0^3 b_1 + 20 c a_{-1} b_{-1} b_0^3 b_1 \\
&+ 160\alpha^5 a_{-1} b_{-1} b_0^3 b_1 - 1620\alpha a_{-1}^2 a_0 b_{-1} b_1^2 + 1140\alpha^3 a_{-1} a_0 b_{-1}^2 b_1^2 \\
&- 2 c a_0 b_{-1}^3 b_1^2 + 1682\alpha^5 a_0 b_{-1}^3 b_1^2 - 900\alpha a_{-1}^3 b_0 b_1^2 + 1470\alpha^3 a_{-1}^2 b_{-1} b_0 b_1^2 \\
&+ 30 c a_{-1} b_{-1}^2 b_0 b_1^2 - 1290\alpha^5 a_{-1} b_{-1}^2 b_0 b_1^2 = 0,
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
&360\alpha a_1^3 b_{-1}^3 + 2160\alpha a_0 a_1^2 b_{-1}^2 b_0 + 1080\alpha a_0^2 a_1 b_{-1} b_0^2 \\
&+ 1080\alpha a_{-1} a_1^2 b_{-1} b_0^2 + 720\alpha^3 a_1^2 b_{-1}^2 b_0^2 + 180\alpha^3 a_0 a_1 b_{-1} b_0^3 \\
&- 6 c a_1 b_{-1} b_0^4 + 6\alpha^5 a_1 b_{-1} b_0^4 + 1080\alpha a_0^2 a_1 b_{-1}^2 b_1 + 1080\alpha a_{-1} a_1^2 b_{-1}^2 b_1 \\
&- 720\alpha^3 a_1^2 b_{-1}^3 b_1 - 3060\alpha^3 a_0 a_1 b_{-1}^2 b_0 b_1 - 1080\alpha a_{-1} a_0^2 b_0^2 b_1 \\
&- 1080\alpha a_{-1}^2 a_1 b_0^2 b_1 - 36 c a_1 b_{-1}^2 b_0^2 b_1 - 414\alpha^5 a_1 b_{-1}^2 b_0^2 b_1 \\
&- 180\alpha^3 a_{-1} a_0 b_0^3 b_1 + 6 c a_{-1} b_0^4 b_1 - 6\alpha^5 a_{-1} b_0^4 b_1 - 1080\alpha a_{-1} a_0^2 b_{-1} b_1^2 \\
&- 1080\alpha a_{-1}^2 a_1 b_{-1} b_1^2 - 12 c a_1 b_{-1}^3 b_1^2 + 2112\alpha^5 a_1 b_{-1}^3 b_1^2 \\
&- 4 c a_{-1} b_0^3 b_1 + 5\alpha^5 a_{-1} b_0^3 b_1 + 180\alpha a_{-1}^2 a_0 b_1^2 - 2160\alpha a_{-1}^2 a_0 b_0 b_1^2 \\
&+ 3060\alpha^3 a_{-1} a_0 b_{-1} b_0 b_1^2 - 720\alpha^3 a_1^2 b_0^2 b_1^2 36 c a_{-1} b_{-1} b_0^2 b_1^2 \\
&+ 414\alpha^5 a_{-1} b_{-1} b_0^2 b_1^2 - 360\alpha a_{-1}^3 b_1^3 + 720\alpha^3 a_{-1}^2 b_{-1} b_1^3 + 12 c a_{-1} b_{-1}^2 b_1^3 \\
&- 2112\alpha^5 a_{-1} b_{-1}^2 b_1^3 = 0,
\end{aligned} \tag{4.21}$$

$$\begin{aligned}
&900\alpha a_1^3 b_{-1}^2 b_0 + 1620\alpha a_0 a_1^2 b_{-1} b_0^2 + 180\alpha a_0^2 a_1 b_0^3 \\
&+ 180\alpha a_{-1} a_1^2 b_0^3 + 330\alpha^3 a_1^2 b_{-1} b_0^3 + 30\alpha^3 a_0 a_1 b_0^4 \\
&- c a_1 b_0^5 + \alpha^5 a_1 b_0^5 + 1620\alpha a_0 a_1^2 b_{-1}^2 b_1 + 1080\alpha a_0^2 a_1 b_{-1} b_0 b_1 \\
&+ 1080\alpha a_{-1} a_1^2 b_{-1} b_0 b_1 - 1470\alpha^3 a_{-1}^2 a_0 b_{-1}^2 b_0 b_1 - 180\alpha a_0^3 b_0^2 b_1 \\
&- 1080\alpha a_{-1} a_0 a_1 b_0^2 b_1 - 960\alpha^3 a_0 a_1 b_{-1} b_0^2 b_1 - 30\alpha^3 a_0^2 b_0^3 b_1 \\
&- 180\alpha^3 a_{-1} a_1 b_0^3 b_1 - 20 c a_1 b_{-1} b_0^3 b_1 - 160\alpha^5 a_1 b_{-1} b_0^3 b_1 \\
&+ c a_0 b_0^4 b_1 - 180\alpha a_0^3 b_{-1} b_1^2 - 1080\alpha a_{-1} a_0 a_1 b_{-1} b_1^2 \\
&- 1140\alpha^3 a_0 a_1^2 b_{-1}^2 b_1^2 - 1620\alpha^3 a_{-1} a_0^2 b_0 b_1^2 - 1620\alpha a_{-1}^2 a_1 b_0 b_1^2 \\
&+ 1050\alpha^3 a_0^2 b_{-1} b_0 b_1^2 + 1740\alpha^3 a_{-1} a_1 b_{-1} b_0 b_1^2 - 30 c a_1 b_{-1}^2 b_0 b_1^2 \\
&+ 1290\alpha^5 a_1 b_{-1}^2 b_0 b_1^2 + 240\alpha^3 a_{-1} a_0 b_0^2 b_1^2 \\
&+ 6 c a_0 b_{-1} b_0^2 b_1^2 + 174\alpha^5 a_0 b_{-1} b_0^2 b_1^2 + 14 c a_{-1} b_0^3 b_1^2 \\
&- 14\alpha^5 a_{-1} b_0^3 b_1^2 - 900\alpha a_{-1}^2 a_0 b_1^3 + 1800\alpha^3 a_{-1} a_0 b_{-1} b_1^3 \\
&+ 2 c a_0 b_{-1}^2 b_1^3 - 1682\alpha^5 a_0 b_{-1}^2 b_1^3 - 930\alpha^3 a_1^2 b_0 b_1^3 + 28 c a_{-1} b_{-1} b_0 b_1^3 \\
&+ 392\alpha^5 a_{-1} b_1 b_0 b_1^3 = 0,
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
&720\alpha a_1 b_0 b_{-1} b_0^2 + 360\alpha a_0 a_1^2 b_0^3 + 60\alpha^3 a_1^2 b_0^4 \\
&+ 720\alpha a_1^3 b_{-1}^2 b_1 + 2160\alpha a_0^2 a_1^2 b_{-1} b_0 b_1 - 600\alpha^3 a_1^2 b_{-1} b_0 b_1 \\
&- 180\alpha^3 a_0 a_1 b_{-1} b_0^3 b_1 - 4 c a_1 b_0^4 b_1 - 26\alpha^5 a_1 b_0^4 b_1 - 960\alpha^3 a_1^2 b_{-1}^2 b_1^2 \\
&- 360\alpha a_0^3 b_0 b_1^2 - 2160\alpha a_{-1} a_0 a_1 b_0 b_1^2 + 420\alpha^3 a_0 a_1 b_{-1} b_0 b_1^2 \\
&+ 120\alpha^3 a_0^2 b_0^2 b_1^2 - 120\alpha^3 a_{-1} a_1 b_0^2 b_1^2 - 24 c a_1 b_{-1} b_0^2 b_1^2 \\
&+ 474\alpha^5 a_1 b_{-1} b_0^2 b_1^2 + 4 c a_0 b_0^3 b_1^2 + 26\alpha^5 a_0 b_0^3 b_1^2 - 720\alpha a_{-1} a_0^2 b_1^3 \\
&- 720\alpha a_{-1}^2 a_1 b_1^3 + 840\alpha^3 a_0^2 b_{-1} b_1^3 + 1440\alpha^3 a_{-1} a_1 b_{-1} b_1^3 - 8 c a_1 b_{-1}^2 b_1^3 \\
&- 832\alpha^5 a_0 b_{-1}^2 b_1^3 - 540\alpha^3 a_{-1} a_0 b_0 b_1^3 + 8 c a_0 b_{-1} b_0 b_1^3 - 428\alpha^5 a_0 b_{-1} b_0 b_1^3 \\
&+ 16 c a_{-1} b_0^2 b_1^3 - 46\alpha^5 a_{-1} b_0^2 b_1^3 - 480\alpha^3 a_{-1}^2 b_1^4 + 8 c a_{-1} b_{-1} b_1^4 \\
&+ 832\alpha^5 a_{-1} b_{-1} b_1^4 = 0,
\end{aligned} \tag{4.23}$$

$$\begin{aligned}
&180\alpha a_1^3 b_0^3 + 1080\alpha a_1^3 b_{-1} b_0 b_1 + 540\alpha a_0 a_1^2 b_0^2 b_1 - 90\alpha^3 a_1^2 b_0^3 b_1 \\
&+ 540\alpha a_0 a_1^2 b_{-1} b_1^2 - 540\alpha a_0^2 a_1 b_0 b_1^2 - 540\alpha a_{-1} a_1^2 b_0 b_1^2 \\
&- 450\alpha^3 a_1^2 b_{-1} b_0 b_1^2 - 6ca_1 b_0^3 b_1^2 + 66\alpha^5 a_1 b_0^3 b_1^2 - 180\alpha a_0^3 b_1^3 \\
&- 1080\alpha a_{-1} a_0 a_1 b_1^3 + 1080\alpha^3 a_0 a_1 b_{-1} b_1^3 + 90\alpha^3 a_0^2 b_0 b_1^3 \\
&- 180\alpha^3 a_{-1} a_1 b_0 b_1^3 - 12ca_1 b_{-1} b_0 b_1^3 - 288\alpha^5 a_1 b_{-1} b_0 b_1^3 + 6ca_0 b_0^2 b_1^3 \\
&+ 30\alpha^3 a_0^2 b_1^3 + 120\alpha^3 a_{-1} a_1 b_1^3 - 4ca_1 b_{-1} b_1^3 \\
&- 66\alpha^5 a_0 b_0^2 b_1^3 - 450\alpha^3 a_{-1} a_0 b_1^4 3ca_0 b_{-1} b_1^4 + 237\alpha^5 a_0 b_{-1} b_1^4 \\
&+ 9ca_{-1} b_0 b_1^4 + 51\alpha^5 a_{-1} b_0 b_1^4 = 0,
\end{aligned} \tag{4.24}$$

$$\begin{aligned}
&360\alpha a_1^3 b_0^2 b_1 + 360\alpha a_1^3 b_{-1} b_1^2 - 120\alpha^3 a_1^2 b_0^2 b_1^2 - 360\alpha a_0^2 a_1 b_1^3 \\
&- 360\alpha a_{-1} a_1^2 b_1^3 + 240\alpha^3 a_1^2 b_{-1} b_1^3 + 180\alpha^3 a_0 a_1 b_0 b_1^3 - 4ca_1 b_0^2 b_1^3 \\
&- 26\alpha^5 a_1 b_0^2 b_1^3 - 60\alpha^3 a_0^2 b_1^4 - 240\alpha^3 a_{-1} a_1 b_1^4 - 2ca_1 b_{-1} b_1^4 \\
&+ 32\alpha^5 a_1 b_{-1} b_1^4 + 4ca_0 b_0 b_1^4 + 26\alpha^5 a_0 b_0 b_1^4 + 2ca_{-1} b_1^5 - 32\alpha^5 a_{-1} b_1^5 = 0,
\end{aligned} \tag{4.25}$$

$$\begin{aligned}
&180\alpha a_1^3 b_0 b_1^2 - 180\alpha a_0 a_1^2 b_1^3 + 30\alpha^3 a_1^2 b_0 b_1^3 - 30\alpha^3 a_0 a_1 b_1^4 \\
&- ca_1 b_0 b_1^4 + \alpha^5 a_1 b_0 b_1^4 + ca_0 b_1^5 - \alpha^5 a_0 b_1^5 = 0.
\end{aligned} \tag{4.26}$$

Sistemi

$$a_{-1},\ a_0,\ a_1,\ b_{-1},\ b_0, \tag{4.27}$$

için çözülürse aşağıdaki iki farklı durum elde edilir.

Durum 1

$$\begin{aligned}
a_{-1} = &\frac{2cb_0^4}{15\alpha^3 b_0^2 - \sqrt{5} b_0 \sqrt{\alpha(4c+\alpha^5) b_0^2} 24 b_1} \\
&- \frac{7\alpha^5 b_0^4}{15\alpha^3 b_0^2 - \sqrt{5} b_0 \sqrt{\alpha(4c+\alpha^5) b_0^2} 24 b_1} \\
&- \frac{\sqrt{5}\alpha^2 b_0^4 \sqrt{\alpha(4c+\alpha^5)}}{15\alpha^3 b_0^2 - \sqrt{5} b_0^2 \sqrt{\alpha(4c+\alpha^5)} 24 b_1},
\end{aligned} \tag{4.28}$$

$$a_0 = \frac{25\alpha^3 b_0 - \sqrt{5}\sqrt{4c\alpha b_0^2 + \alpha^6 b_0^2}}{60\alpha}, \tag{4.29}$$

$$a_1 = \frac{-2cb_0^2 b_1 + 7\alpha^5 b_0^2 b_1 + \sqrt{5}\alpha^2 b_0 \sqrt{\alpha(4c+\alpha^5)b_0^2}\, b_1}{6\big(-15\alpha^3 b_0^2 + \sqrt{5} b_0 \sqrt{\alpha(4c+\alpha^5)b_0^2}\big)}, \tag{4.30}$$

$$b_{-1} = \frac{b_0^2}{4b_1}. \tag{4.31}$$

Bu halde genel çözüm

$$A(x,t) = -\frac{5\alpha^3 + \frac{\sqrt{5}\sqrt{\alpha(4c+\alpha^5)b_0^2}}{b_0} - \frac{120e^{ct}\alpha^3 b_0 b_1}{4e^{ct}b_0 b_1 + e^{x\alpha}(b_0^2+4b_1^2)}}{60\alpha} \tag{4.32}$$

şeklinde olur.

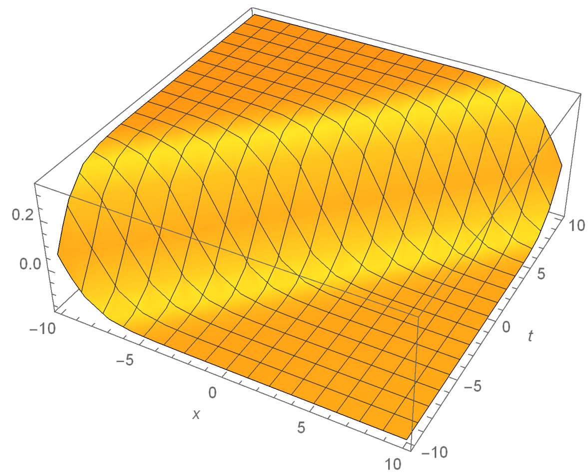
Ayrıca,

$$b_0 = b_1 = \alpha = c = 1 \tag{4.33}$$

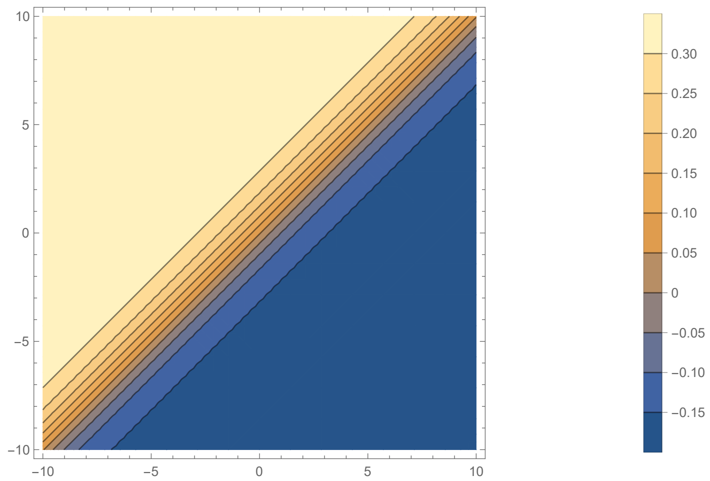
olacak şekilde seçilirse bir özel çözüm

$$A(x,t) = \frac{8 - 5e^{-t+x}}{24 + 30e^{-t+x}} \tag{4.34}$$

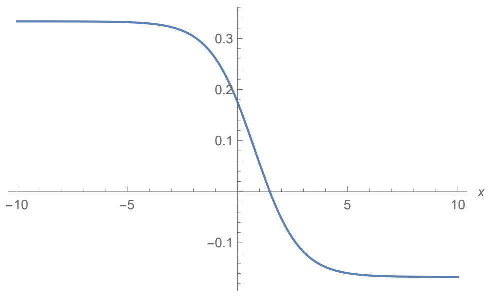
biçiminde olur.



Şekil 4.1: (**??**) denkleminin 3D grafiği $-10 \le t \le 10$ ve $-10 \le x \le 10$.



Şekil 4.2: (**??**) denkleminin 3D contour grafiği $-10 \le t \le 10$ ve $-10 \le x \le 10$.



Şekil 4.3: (**??**) denkleminin 2D grafiği $-10 \le t \le 10$.

Durum 2

$$a_{-1} = \frac{2cb_0^4}{15\alpha^3 b_0^2 + \sqrt{5}b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}24b_1} - \frac{7\alpha^5 b_0^4}{15\alpha^3 b_0^2 + \sqrt{5}b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}24b_1} + \frac{\sqrt{5}\alpha^2 b_0^4\sqrt{\alpha(4c+\alpha^5)}}{15\alpha^3 b_0^2 + \sqrt{5}b_0^2\sqrt{\alpha(4c+\alpha^5)}24b_1}, \tag{4.35}$$

$$a_0 = \frac{25\alpha^3 b_0 + \sqrt{5}\sqrt{4c\alpha b_0^2 + \alpha^6 b_0^2}}{60\alpha}, \tag{4.36}$$

$$a_1 = \frac{2cb_0^2 b_1 + 7\alpha^5 b_0^2 b_1 + \sqrt{5}\alpha^2 b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}b_1}{6\left(15\alpha^3 b_0^2 + \sqrt{5}b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}\right)}, \tag{4.37}$$

$$b_{-1} = \frac{b_0^2}{4b_1}. \tag{4.38}$$

Bu halde genel çözüm

$$A(x,t) = -\frac{5\alpha^3 + \frac{\sqrt{5}\sqrt{\alpha(4c+\alpha^5)b_0^2}}{b_0} + \frac{120e^{ct}\alpha^3 b_0 b_1}{4e^{ct}b_0 b_1 + e^{x\alpha}(b_0^2+4b_1^2)}}{60\alpha} \tag{4.39}$$

şeklinde olur. Ayrıca,

$$b_0 = b_1 = \alpha = c = 1 \tag{4.40}$$

olacak şekilde seçilirse, özel çözüm

$$A(x,t) = \frac{2}{4+5e^{-t+x}} \tag{4.41}$$

biçiminde olur.

Durum 3: Periyodik Çözüm

$$
\begin{aligned}
a_0 &= \frac{25\alpha^3 b_0 - \sqrt{5}\sqrt{4c\alpha b_0^2 + \alpha^6 b_0^2}}{60\alpha},\\
a_1 &= \frac{-2cb_0^2 b_1 + 7\alpha^5 b_0^2 b_1 + \sqrt{5}\alpha^2 b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}\,b_1}{6\big(-15\alpha^3 b_0^2 + \sqrt{5}b_0\sqrt{\alpha(4c+\alpha^5)b_0^2}\big)},\\
b_{-1} &= \frac{b_0^2}{4b_1}.
\end{aligned} \tag{4.42}
$$

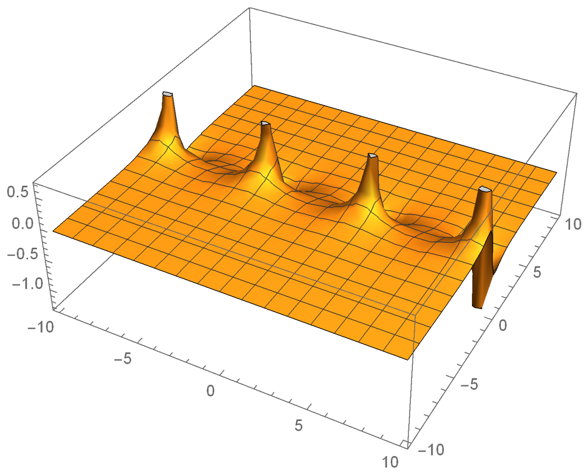
Bu halde, CDGSK denkleminin özel periyodik çözümleri,

$$b_0 = b_1 = c = 1, \alpha = i \tag{4.43}$$

olarak seçilirse

$$A = \pm 0,0596467 - \frac{2 + \frac{5}{2}e^{-t}\cos x}{4 + \frac{25e^{-2t}}{4} + 10e^{-t}\cos x} \tag{4.44}$$

biçiminde olur.



Şekil 4.4: (**??**) denkleminin 3D grafiği $-10 \le t \le 10$ ve $-10 \le x \le 10$.

4.2. KdV Benzeri Denklem

$$U_{xxx} + \alpha_2 U_{yyy} + \alpha_3 U U_x + \alpha_4 U_t = 0 \tag{4.45}$$

denklemine

$$\xi = ax + dy - ct,\ U(x,y,t) = U(\xi) \tag{4.46}$$

dönüşümü tatbik edilirse,

$$(a^3 + \alpha_2 d^3)U''' + \alpha_3 a U U' - \alpha_4 c U' = 0 \tag{4.47}$$

bayağı diferansiyel denklemi elde edilir. Ayrıca, (**??**) denklemi integre edilir ve integral sabiti sıfır alınırsa

$$(a^3 + \alpha_2 d^3)U'' + \alpha_3 a U^2 - \alpha_4 c U = 0 \tag{4.48}$$

bulunur. (**??**) denklemi yani çözüm fonksiyonu olarak kabul edilen c, d, p ve q sayılarını bulabilmek için yönteme göre dengeleme işlemi uygulanmalıdır. Dengeleme (**??**) denklemine uygulanırsa

$$\begin{aligned} U^2 &= \frac{a_{-c}^2 e^{-2c\xi} + \cdots}{b_{-p}^2 e^{-2p\xi} + \cdots}, \\ U'' &= \frac{G e^{-(c+3p)\xi} + \cdots}{E e^{-4p\xi} + \cdots} \end{aligned} \tag{4.49}$$

denklemlerinden

$$2c + 2p = c + 3p,\ p = c \tag{4.50}$$

ve

$$\begin{aligned} U^2 &= \frac{\cdots + a_d^2 e^{2d\xi}}{\cdots + b_q^2 e^{2q\xi}}, \\ U'' &= \frac{\cdots + H e^{(d+3q)\xi}}{\cdots + F e^{4q\xi}} \end{aligned} \tag{4.51}$$

denklemlerinden

$$2d + 2q = d + 3q, \ d = q \tag{4.52}$$

elde edilir. Şimdi, $p = c = d = q = 1$ alalım. Bu durumda,

$$U(\xi) = \frac{a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi}} \tag{4.53}$$

elde edilir. Burada amacımız,

$$a_{-1}, \ a_0, \ a_1, \ b_{-1}, \ b_0, \ b_1 \tag{4.54}$$

katsayılarını bulmaktır. Ayrıca, (**??**),(**??**) ve (**??**) denklemleri (**??**) denkleminde yerine yazılır ve $n = 1, \ldots, 7$ olmak üzere eksponansiyel ifadesinin katsayıları dikkate alınırsa aşağıdaki sistem elde edilir.

$$\begin{aligned} &a^3 a_0 b_{-1}^3 - a^3 a_{-1} b_{-1}^2 b_0 + d^3 a_0 b_{-1}^3 \alpha_2 - d^3 a_{-1} b_{-1}^2 b_0 \alpha_2 \\ &+ a a_{-1} a_0 b_{-1}^2 \alpha_3 - a a_{-1}^2 b_{-1} b_0 \alpha_3 - c a_0 b_{-1}^3 \alpha_4 + c a_{-1} b_{-1}^2 b_0 \alpha_4 = 0, \end{aligned} \tag{4.55}$$

$$\begin{aligned} &8a^3 a_1 b_{-1}^3 - 4a^3 a_0 b_{-1}^2 b_0 + 4a^3 a_{-1} b_{-1} b_0^2 - 8a^3 a_{-1} b_{-1}^2 b_1 \\ &+ 8d^3 a_1 b_{-1}^3 \alpha_2 - 4d^3 a_0 b_{-1}^2 b_0 \alpha_2 + 4d^3 a_{-1} b_{-1} b_0^2 \alpha_2 - 8d^3 a_{-1} b_{-1}^2 b_1 \alpha_2 \\ &+ a a_0^2 b_{-1}^2 \alpha_3 + 2a a_{-1} a_1 b_{-1}^2 \alpha_3 - a a_{-1}^2 b_0^2 \alpha_3 - 2a a_{-1}^2 b_{-1} b_1 \alpha_3 \\ &- 2c a_1 b_{-1}^3 \alpha_4 - 2c a_0 b_{-1}^2 b_0 \alpha_4 + 2c a_{-1} b_{-1} b_0^2 \alpha_4 + 2c a_{-1} b_{-1}^2 b_1 \alpha_4 = 0, \end{aligned} \tag{4.56}$$

$$\begin{aligned} &5a^3 a_1 b_{-1}^2 b_0 + a^3 a_0 b_{-1} b_0^2 - a^3 a_{-1} b_0^3 - 23a^3 a_0 b_{-1}^2 b_1 \\ &+ 18a^3 a_{-1} b_{-1} b_0 b_1 + 5d^3 a_1 b_{-1}^2 b_0 \alpha_2 + d^3 a_0 b_{-1} b_0^2 \alpha_2 - d^3 a_{-1} b_0^3 \alpha_2 \\ &- 23d^3 a_0 b_{-1}^2 b_1 \alpha_2 + 18d^3 a_{-1} b_{-1} b_0 b_1 \alpha_2 + 3a a_0 a_1 b_{-1}^2 \alpha_3 + a a_0^2 b_{-1} b_0 \alpha_3 \\ &+ 2a a_{-1} a_1 b_{-1} b_0 \alpha_3 - a a_{-1} a_0 b_0^2 \alpha_3 - 2a a_{-1} a_0 b_{-1} b_1 \alpha_3 - 3a a_{-1}^2 b_0 b_1 \alpha_3 \\ &- 5c a_1 b_{-1}^2 b_0 \alpha_4 - c a_0 b_{-1} b_0^2 \alpha_4 + c a_{-1} b_0^3 \alpha_4 - c a_0 b_{-1}^2 b_1 \alpha_4 \\ &+ 6c a_{-1} b_{-1} b_0 b_1 \alpha_4 = 0, \end{aligned} \tag{4.57}$$

$$\begin{aligned}
&4a^3a_1b_{-1}b_0^2 - 32a^3a_1b_{-1}^2b_1 - 4a^3a_{-1}b_0^2b_1 + 32a^3a_{-1}b_{-1}b_1^2 \\
&+ 4d^3a_1b_{-1}b_0^2\alpha_2 - 32d^3a_1b_{-1}^2b_1\alpha_2 - 4d^3a_{-1}b_0^2b_1\alpha_2 + 32d^3a_{-1}b_{-1}b_1^2\alpha_2 \\
&+ 2aa_1^2b_{-1}^2\alpha_3 + 4aa_0a_1b_{-1}b_0\alpha_3 - 4aa_{-1}a_0b_0b_1\alpha_3 - 2aa_{-1}^2b_1^2\alpha_3 \\
&- 4ca_1b_{-1}b_0^2\alpha_4 - 4ca_1b_{-1}^2b_1\alpha_4 + 4ca_{-1}b_0^2b_1\alpha_4 + 4ca_{-1}b_{-1}b_1^2\alpha_4 = 0,
\end{aligned} \tag{4.58}$$

$$\begin{aligned}
&a^3a_1b_0^3 - 18a^3a_1b_{-1}b_0b_1 - a^3a_0b_0^2b_1 + 23a^3a_0b_{-1}b_1^2 \\
&- 5a^3a_{-1}b_0b_1^2 + d^3a_1b_0^3\alpha_2 - 18d^3a_1b_{-1}b_0b_1\alpha_2 - d^3a_0b_0^2b_1\alpha_2 \\
&+ 23d^3a_0b_{-1}b_1^2\alpha_2 - 5d^3a_{-1}b_0b_1^2\alpha_2 + 3aa_1^2b_{-1}b_0\alpha_3 + aa_0a_1b_0^2\alpha_3 \\
&+ 2aa_0a_1b_{-1}b_1\alpha_3 - aa_0^2b_0b_1\alpha_3 - 2aa_{-1}a_1b_0b_1\alpha_3 - 3aa_{-1}a_0b_1^2\alpha_3 \\
&- ca_1b_0^3\alpha_4 - 6ca_1b_{-1}b_0b_1\alpha_4 + ca_0b_0^2b_1\alpha_4 + ca_0b_{-1}b_1^2\alpha_4 \\
&+ 5ca_{-1}b_0b_1^2\alpha_4 = 0,
\end{aligned} \tag{4.59}$$

$$\begin{aligned}
&- 4a^3a_1b_0^2b_1 + 8a^3a_1b_{-1}b_1^2 + 4a^3a_0b_0b_1^2 - 8a^3a_{-1}b_1^3 \\
&- 4d^3a_1b_0^2b_1\alpha_2 + 8d^3a_1b_{-1}b_1^2\alpha_2 + 4d^3a_0b_0b_1^2\alpha_2 - 8d^3a_{-1}b_1^3\alpha_2 \\
&+ aa_1^2b_0^2\alpha_3 + 2aa_1^2b_{-1}b_1\alpha_3 - aa_0^2b_1^2\alpha_3 - 2aa_{-1}a_1b_1^2\alpha_3 \\
&- 2ca_1b_0^2b_1\alpha_4 - 2ca_1b_{-1}b_1^2\alpha_4 + 2ca_0b_0b_1^2\alpha_4 + 2ca_{-1}b_1^3\alpha_4 = 0,
\end{aligned} \tag{4.60}$$

$$\begin{aligned}
&a^3a_1b_0b_1^2 - a^3a_0b_1^3 + d^3a_1b_0b_1^2\alpha_2 - d^3a_0b_1^3\alpha_2 \\
&+ aa_1^2b_0b_1\alpha_3 - aa_0a_1b_1^2\alpha_3 - ca_1b_0b_1^2\alpha_4 + ca_0b_1^3\alpha_4 = 0.
\end{aligned} \tag{4.61}$$

Sistemi

$$a_{-1},\ a_0,\ a_1,\ b_{-1},\ b_0,\ b_1 \tag{4.62}$$

için çözülürse sadece bir durum oluşur.

Durum 1

$$\begin{aligned}
a_{-1} &= -\frac{b_0^2(a^3 + d^3\alpha_2 - c\alpha_4)}{4ab_1\alpha_3}, \\
a_0 &= \frac{b_0(5a^3 + 5d^3\alpha_2 + c\alpha_4)}{a\alpha_3}, \\
a_1 &= \frac{-a^3b_1 - d^3b_1\alpha_2 + cb_1\alpha_4}{a\alpha_3}, b_{-1} = \frac{b_0^2}{4b_1}.
\end{aligned} \tag{4.63}$$

Bu halde genel çözüm

$$U(x,y,t) = \frac{a^3\left(-1+\frac{24e^{\xi}b_0b_1}{(b_0+2e^{\xi}b_1)^2}\right)+d^3\left(-1+\frac{24e^{\xi}b_0b_1}{(b_0+2e^{\xi}b_1)^2}\right)\alpha_2+c\alpha_4}{a\alpha_3} \tag{4.64}$$

şeklinde olur. Ayrıca,

$$a = c = d = y = \alpha_2 = \alpha_3 = \alpha_4 = b_0 = b_1 = 1 \tag{4.65}$$

olmak üzere

$$U(x,t) = -\frac{e^{2t} - 44e^{1+t+x} + 4e^{2+2x}}{(e^t + 2e^{1+x})^2} \tag{4.66}$$

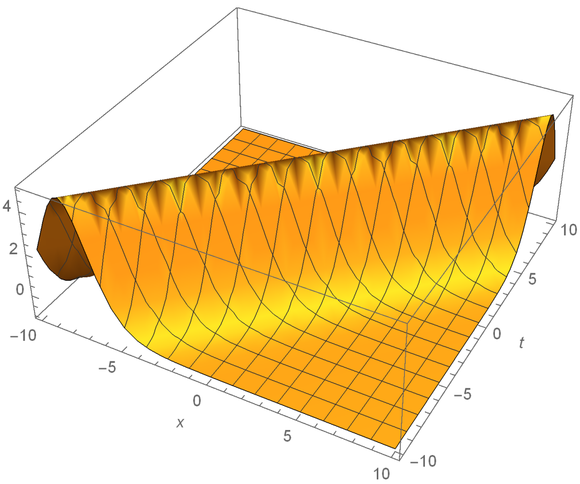
özel çözümü elde edilir. Son olarak, bilinen KdV denklemi için

$$a = 1,\ c = 1,\ d = 0,\ \alpha_2 = 0,\ \alpha_3 = -6,\ \alpha_4 = 1,\ b_0 = 1,\ b_1 = 1 \tag{4.67}$$

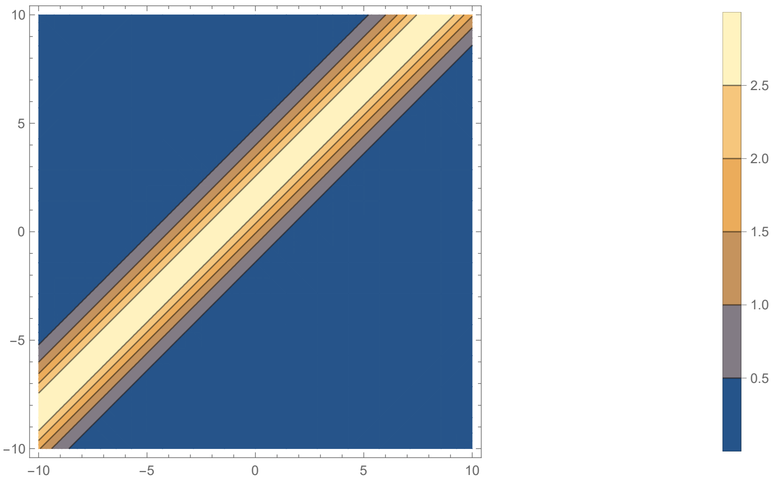
değerleri dikkate alınırsa

$$U(x,t) = -\frac{4e^{t+x}}{(e^t + 2e^x)^2} \tag{4.68}$$

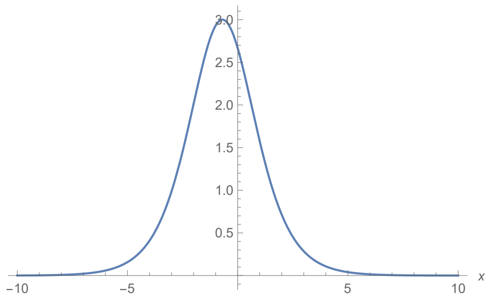
çözümü elde edilir.



Şekil 4.5: (**??**) denkleminin 3D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



Şekil 4.6: (**??**) denkleminin 3D contour grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



Şekil 4.7: (**??**) denkleminin 2D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.

4.3. JM Denklemi

$$V_{xxxy} + A_2V_yV_{xx} + A_3V_xV_{xy} + A_4V_{yt} - A_5V_{xz} = 0 \tag{4.69}$$

denklemine

$$\xi = c_1x + c_2y + c_3z - c_4t,\ V(x, y, z, t) = V(\xi) \tag{4.70}$$

dönüşümü tatbik edilirse

$$\alpha = c_1^3c_2,\ \beta = A_2c_1^2c_2 + A_3c_1^2c_2,\ \gamma = A_4c_2c_4 + A_5c_1c_3 \tag{4.71}$$

olmak üzere

$$\alpha V^{(4)} + \beta V'V'' + \gamma V'' = 0 \tag{4.72}$$

denklemi elde edilir. (**??**) denklemi yani çözüm fonksiyonu olarak kabul edilen c, d, p ve q sayılarını bulabilmek için yönteme göre dengeleme işlemi uygulanmalıdır. Dengeleme (**??**) denklemine uygulanırsa

$$\begin{aligned} V^{(4)} &= \frac{Me^{-(c+15p)\xi} + \cdots}{Pe^{-16p\xi} + \cdots}, \\ V'V'' &= \frac{Ge^{-(2c+4p)\xi} + \cdots}{Ee^{-6p\xi} + \cdots} \end{aligned} \tag{4.73}$$

denklemlerinden

$$c + 15p = 2c + 14p,\ p = c \tag{4.74}$$

ve

$$\begin{aligned} V^{(4)} &= \frac{\cdots + Ne^{(d+15q)\xi}}{\cdots + Re^{16q\xi}}, \\ V'V'' &= \frac{\cdots + He^{(2d+4q)\xi}}{\cdots + Fe^{6q\xi}} \end{aligned} \tag{4.75}$$

denklemlerinden

$$d + 15q = 2d + 14q,\ d = q \tag{4.76}$$

elde edilir. Şimdi, $p = c = d = q = 1$ alalım. Bu durumda,

$$V(\xi) = \frac{a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + b_1 e^{\xi}} \tag{4.77}$$

elde edilir. Burada amaç,

$$a_{-1},\ a_0,\ a_1,\ b_{-1},\ b_0,\ b_1 \tag{4.78}$$

katsayılarını bulmaktır. Ayrıca, (**??**)-(**??**) denklemleri (**??**) denkleminde yerine yazılır ve $n = 1, \ldots, 9$ olmak üzere eksponanasiyel ifadesinin katsayıları dikkate alınırsa aşağıda verilen dokuz denklemden oluşan bir sistem elde edilir.

$$\alpha a_0 b_{-1}^4 + \gamma a_0 b_{-1}^4 - \alpha a_{-1} b_{-1}^3 b_0 - \gamma a_{-1} b_{-1}^3 b_0 = 0, \tag{4.79}$$

$$\begin{aligned} &\beta a_0^2 b_{-1}^3 + 16\alpha a_1 b_{-1}^4 + 4\gamma a_1 b_{-1}^4 - 2\beta a_{-1} a_0 b_{-1}^2 b_0 - 11\alpha a_0 b_{-1}^3 b_0 \\ &+ \gamma a_0 b_{-1}^3 b_0 + \beta a_{-1}^2 b_{-1} b_0^2 + 11\alpha a_{-1} b_{-1}^2 b_0^2 - \gamma a_{-1} b_{-1}^2 b_0^2 \\ &- 16\alpha a_{-1} b_{-1}^3 b_1 - 4\gamma a_{-1} b_{-1}^3 b_1 = 0, \end{aligned} \tag{4.80}$$

$$\begin{aligned} &6\beta a_0 a_1 b_{-1}^3 - \beta a_0^2 b_{-1}^2 b_0 - 6\beta a_{-1} a_1 b_{-1}^2 b_0 - \alpha a_1 b_{-1}^3 b_0 + 11 a_1 b_{-1}^3 b_0 \\ &+ 2\beta a_{-1} a_0 b_{-1} b_0^2 + 11\alpha a_0 b_{-1}^2 b_0^2 - \gamma a_0 b_{-1}^2 b_0^2 - \beta a_{-1}^2 b_0^3 \\ &- 11\alpha a_{-1} b_{-1} b_0^3 + \gamma a_{-1} b_{-1} b_0^3 - 6\beta a_{-1} a_0 b_{-1}^2 b_1 - 76\alpha a_0 b_{-1}^3 b_1 \\ &- 4\gamma a_0 b_{-1}^3 b_1 + 6\beta a_{-1}^2 b_{-1} b_0 b_1 + 77\alpha a_{-1} b_{-1}^2 b_0 b_1 - 7\gamma a_{-1} b_{-1}^2 b_0 b_1 = 0, \end{aligned} \tag{4.81}$$

$$\begin{aligned} &8\beta a_1^2 b_{-1}^3 + 2\beta a_0 a_1 b_{-1}^2 b_0 - 2\beta a_{-1} a_1 b_{-1} b_0^2 + 11\alpha a_1 b_{-1}^2 b_0^2 \\ &+ 11\gamma a_1 b_{-1}^2 b_0^2 - \alpha a_0 b_{-1} b_0^3 - \gamma a_0 b_{-1} b_0^3 + \alpha a_{-1} b_0^4 + \gamma a_{-1} b_0^4 \\ &- 7\beta a_0^2 b_{-1}^2 b_1 - 16\beta a_{-1} a_1 b_{-1}^2 b_1 - 176\alpha a_1 b_{-1}^3 b_1 + 4\gamma a_1 b_{-1}^3 b_1 \\ &+ 12\beta a_{-1} a_0 b_{-1} b_0 b_1 + 47\alpha a_0 b_{-1}^2 b_0 b_1 - 13\gamma a_0 b_{-1}^2 b_0 b_1 - 5\beta a_{-1}^2 b_0^2 b_1 \\ &- 58\alpha a_{-1} b_{-1} b_0^2 b_1 + 2\gamma a_{-1} b_{-1} b_0^2 b_1 + 8\beta a_{-1}^2 b_{-1} b_1^2 + 176 a_{-1} b_{-1}^2 b_1^2 \\ &- 4\gamma a_{-1} b_{-1}^2 b_1^2 = 0, \end{aligned} \tag{4.82}$$

$$\begin{aligned}
&10\beta a_1^2 b_{-1}^2 b_0 + 5\alpha a_1 b_{-1} b_0^3 + 5\gamma a_1 b_{-1} b_0^3 - 20\beta a_0 a_1 b_{-1}^2 b_1 \\
&- 115\alpha a_1 b_{-1}^2 b_0 b_1 + 5\gamma a_1 b_{-1}^2 b_0 b_1 - 10\alpha a_0 b_{-1} b_0^2 b_1 - 10\gamma a_0 b_{-1} b_0^2 b_1 \\
&+ 5\alpha a_{-1} b_0^3 b_1 + 5\gamma a_{-1} b_0^3 b_1 + 20\beta a_{-1} a_0 b_{-1} b_1^2 + 230\alpha a_0 b_{-1}^2 b_1^2 \\
&- 10\gamma a_0 b_{-1}^2 b_1^2 - 10\beta a_{-1}^2 b_0 b_1^2 - 115\alpha a_{-1} b_{-1} b_0 b_1^2 + 5\gamma a_{-1} b_{-1} b_0 b_1^2 = 0,
\end{aligned} \tag{4.83}$$

$$\begin{aligned}
&5\beta a_1^2 b_{-1} b_0^2 + \alpha a_1 b_0^4 + \gamma a_1 b_0^4 - 8\beta a_1^2 b_{-1}^2 b_1 - 12\beta a_0 a_1 b_{-1} b_0 b_1 \\
&+ 2\beta a_{-1} a_1 b_0^2 b_1 - 58\alpha a_1 b_{-1} b_0^2 b_1 + 2\gamma a_1 b_{-1} b_0^2 b_1 - \alpha a_0 b_0^3 b_1 \\
&- \gamma a_0 b_0^3 b_1 + 7\beta a_0^2 b_{-1} b_1^2 + 16\beta a_{-1} a_1 b_{-1} b_1^2 + 176\alpha a_1 b_{-1}^2 b_1^2 \\
&- 4\gamma a_1 b_{-1}^2 b_1^2 - 2\beta a_{-1} a_0 b_0 b_1^2 + 47\alpha a_0 b_{-1} b_0 b_1^2 - 13\gamma a_0 b_{-1} b_0 b_1^2 \\
&+ 11\alpha a_{-1} b_0^2 b_1^2 + 11\gamma a_{-1} b_0^2 b_1^2 - 8\beta a_{-1}^2 b_1^3 - 176\alpha a_{-1} b_{-1} b_1^3 \\
&+ 4\gamma a_{-1} b_{-1} b_1^3 = 0,
\end{aligned} \tag{4.84}$$

$$\begin{aligned}
&\beta a_1^2 b_0^3 - 6\beta a_1^2 b_{-1} b_0 b_1 - 2\beta a_0 a_1 b_0^2 b_1 - 11\alpha a_1 b_0^3 b_1 + \gamma a_1 b_0^3 b_1 \\
&+ 6\beta a_0 a_1 b_{-1} b_1^2 + \beta a_0^2 b_0 b_1^2 + 6\beta a_{-1} a_1 b_0 b_1^2 + 77\alpha a_1 b_{-1} b_0 b_1^2 \\
&- 7\gamma a_1 b_{-1} b_0 b_1^2 + 11\alpha a_0 b_0^2 b_1^2 - \gamma a_0 b_0^2 b_1^2 - 6\beta a_{-1} a_0 b_1^3 \\
&- 76\alpha a_0 b_{-1} b_1^3 - 4\gamma a_0 b_{-1} b_1^3 - \alpha a_{-1} b_0 b_1^3 + 11\gamma a_{-1} b_0 b_1^3 = 0,
\end{aligned} \tag{4.85}$$

$$\begin{aligned}
&- \beta a_1^2 b_0^2 b_1 + 2\beta a_0 a_1 b_0 b_1^2 + 11\alpha a_1 b_0^2 b_1^2 - \gamma a_1 b_0^2 b_1^2 - \beta a_0^2 b_1^3 \\
&- 16\alpha a_1 b_{-1} b_1^3 - 4\gamma a_1 b_{-1} b_1^3 - 11\alpha a_0 b_0 b_1^3 + \gamma a_0 b_0 b_1^3 + 16\alpha a_{-1} b_1^4 \\
&+ 4\gamma a_{-1} b_1^4 = 0,
\end{aligned} \tag{4.86}$$

$$-\alpha a_1 b_0 b_1^3 - \gamma a_1 b_0 b_1^3 + \alpha a_0 b_1^4 + \gamma a_0 b_1^4 = 0. \tag{4.87}$$

Sistemi

$$a_{-1},\ a_0,\ a_1,\ b_0,\ \alpha,\ \gamma \tag{4.88}$$

için çözülürse aşağıdaki dört farklı durum elde edilir.

Durum 1

$$\begin{aligned}
&K = a_1 b_{-1} b_0 + a_{-1} b_0 b_1,\\
&a_0 = \frac{K - \sqrt{K^2 - 4b_{-1}b_1(a_1^2 b_{-1}^2 + a_{-1}a_1 b_0^2 - 2a_{-1}a_1 b_{-1}b_1 + a_{-1}^2 b_1^2)}}{2b_{-1}b_1},\\
&\alpha = \frac{\beta(a_1 b_{-1} - a_{-1}b_1)}{12b_{-1}b_1},\ \gamma = \frac{\beta(-a_1 b_{-1} + a_{-1}b_1)}{12b_{-1}b_1}.
\end{aligned} \tag{4.89}$$

Bu halde genel çözüm,

$$\begin{aligned}
&\xi = c_1 x + c_2 y + c_3 z - c_4 t,\ L = e^{xc_1 + yc_2 + (t+z)c_3},\\
&P = \sqrt{b_0^2 - 4b_{-1}b_1},\ N = a_1 b_{-1} - a_{-1}b_1
\end{aligned} \tag{4.90}$$

olmak üzere,

$$V = \frac{2(e^{2tc_3}a_{-1} + e^{2(\xi + c_4 t)}a_1)b_{-1}b_1 + Lb_0(a_1 b_{-1} + a_{-1}b_1) - LNP}{2b_{-1}b_1(e^{2tc_3}b_{-1} + Lb_0 + e^{2(\xi + c_4 t)}b_1)} \tag{4.91}$$

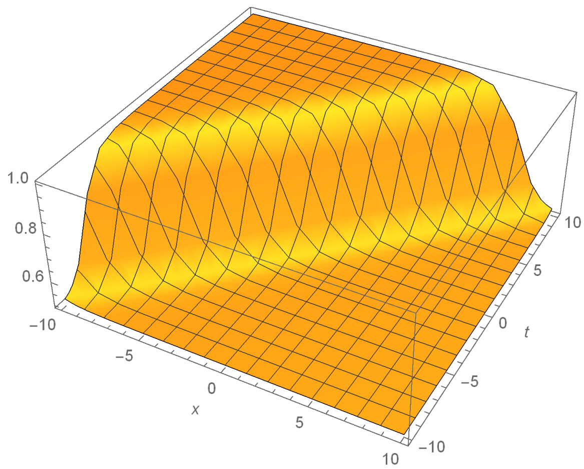
biçiminde olur. Ayrıca,

$$y = z = c_1 = c_2 = c_3 = c_4 = b_{-1} = b_0 = a_1 = a_{-1} = 1,\ b_1 = 2, \tag{4.92}$$

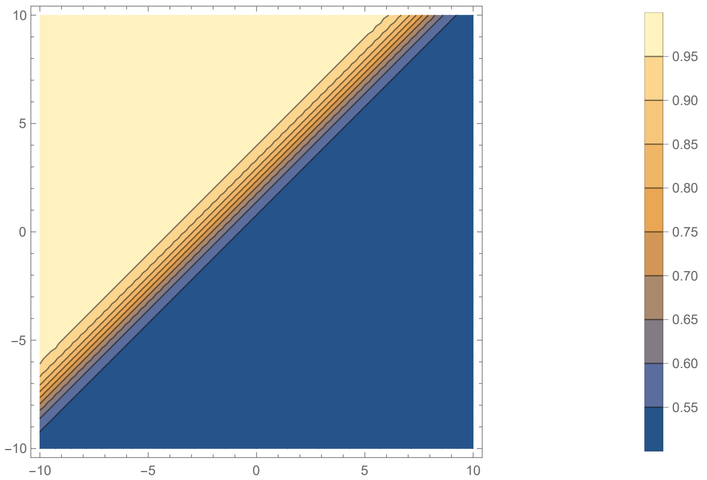
olacak şekilde seçilirse, özel çözüm

$$V = \frac{1}{4}\Big(2 + \frac{2e^{2t} + (1 - i\sqrt{7})e^{2+t+x}}{e^{2t} + e^{2+t+x} + 2e^{4+2x}}\Big) \tag{4.93}$$

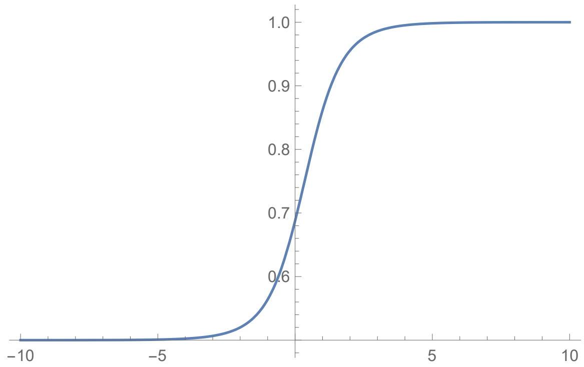
şekinde olur.



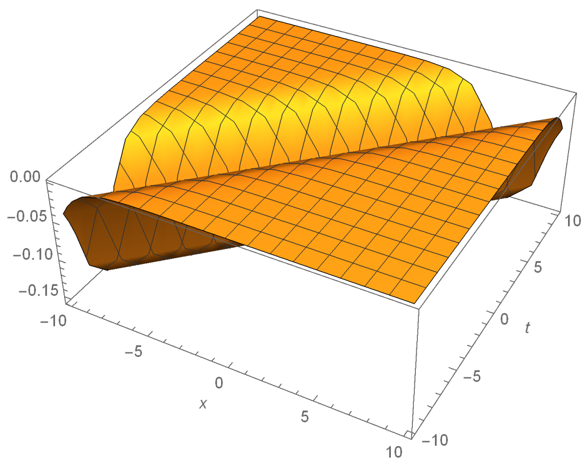
Şekil 4.8: (**??**) denkleminin Reel 3D grafiği $-10 \le t \le 10$ ve $-10 \le x \le 10$.



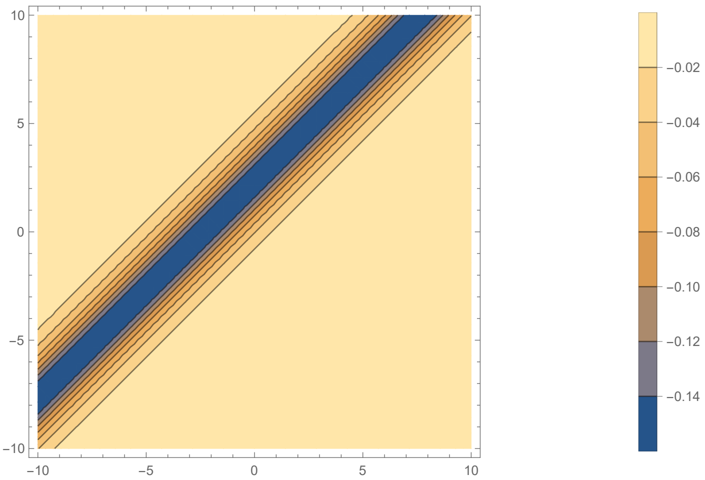
Şekil 4.9: (??) denkleminin Reel 3D contour grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



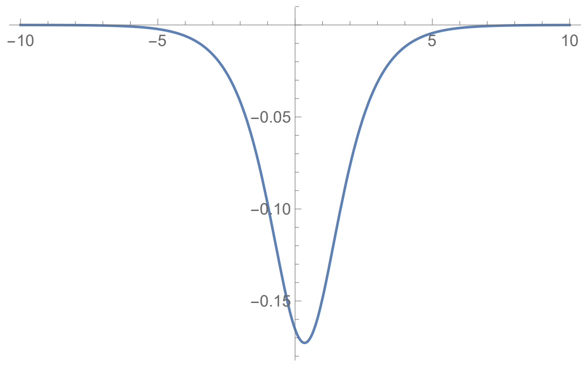
Şekil 4.10: (??) denkleminin Reel 2D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



Şekil 4.11: (??) denkleminin Sanal 3D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



Şekil 4.12: (**??**) denkleminin Sanal 3D contour grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.



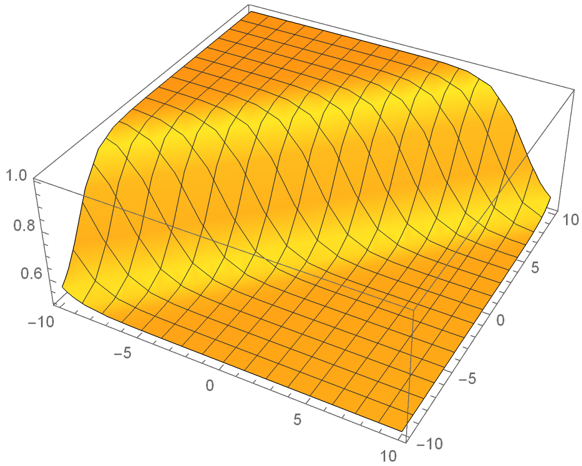
Şekil 4.13: (**??**) denkleminin Sanal 2D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.

$$y = z = c_1 = c_2 = c_3 = c_4 = b_0 = a_1 = 1,\ b_1 = 2,\ b_{-1} = a_{-1} = -1, \tag{4.94}$$

olarak seçilirse

$$V = \frac{1}{2}\left(\frac{2 + e^{2-t+x}}{1 + e^{2-t+x}}\right) \tag{4.95}$$

reel çözümü elde edilir.



Şekil 4.14: (**??**) denkleminin 3D grafiği $-10 \leq t \leq 10$ ve $-10 \leq x \leq 10$.

Durum 2

$$\begin{aligned} &K = a_1 b_{-1} b_0 + a_{-1} b_0 b_1, \\ &a_0 = \frac{K + \sqrt{K^2 - 4b_{-1}b_1(a_1^2 b_{-1}^2 + a_{-1}a_1 b_0^2 - 2a_{-1}a_1 b_{-1}b_1 + a_{-1}^2 b_1^2)}}{2b_{-1}b_1}, \\ &\alpha = \frac{\beta(a_1 b_{-1} - a_{-1}b_1)}{12b_{-1}b_1}, \gamma = \frac{\beta(-a_1 b_{-1} + a_{-1}b_1)}{12b_{-1}b_1}. \end{aligned} \tag{4.96}$$

Bu halde genel çözüm,

$$\begin{aligned} &\xi = c_1 x + c_2 y + c_3 z - c_4 t, \ L = e^{xc_1 + yc_2 + (t+z)c_3}, \\ &P = \sqrt{b_0^2 - 4b_{-1}b_1}, \ N = a_1 b_{-1} - a_{-1}b_1 \end{aligned} \tag{4.97}$$

olmak üzere,

$$V = \frac{2(e^{2tc_3}a_{-1} + e^{2(\xi + c_4 t)}a_1)b_{-1}b_1 + Lb_0(a_1 b_{-1} + a_{-1}b_1) + LNP}{2b_{-1}b_1(e^{2tc_3}b_{-1} + Lb_0 + e^{2(\xi + c_4 t)}b_1)} \tag{4.98}$$

biçiminde olur. Ayrıca,

$$y = z = c_1 = c_2 = c_3 = c_4 = b_{-1} = b_0 = a_1 = a_{-1} = 1, \ b_1 = 2, \tag{4.99}$$

olacak şekilde seçilirse, özel çözüm

$$V = \frac{1}{4}(2 + \frac{2e^{2t} + \left(1 + i\sqrt{7}\right)e^{2+t+x}}{e^{2t} + e^{2+t+x} + 2e^{4+2x}}) \tag{4.100}$$

şekinde olur.

Durum 3

$$a_0 = \frac{-a_1 b_{-1} - a_{-1} b_1}{\sqrt{b_{-1}}\sqrt{b_1}},\ b_0 = -2\sqrt{b_{-1}}\sqrt{b_1},$$
$$\alpha = \frac{\beta(a_1 b_{-1} - a_{-1} b_1)}{12 b_{-1} b_1},\ \gamma = \frac{\beta(-a_1 b_{-1} + a_{-1} b_1)}{12 b_{-1} b_1}. \tag{4.101}$$

Bu halde genel çözüm,

$$V = \frac{a_1 + \frac{e^{tc_3}(-a_1 b_{-1} + a_{-1} b_1)}{e^{tc_3} b_{-1} - e^{xc_1 + yc_2 + zc_3}\sqrt{b_{-1}}\sqrt{b_1}}}{b_1} \tag{4.102}$$

biçiminde olur. Ayrıca,

$$y = z = c_1 = c_2 = c_3 = c_4 = a_1 = a_{-1} = 1, b_{-1} = i,\ b_1 = -i \tag{4.103}$$

olacak şekilde seçilirse, özel çözüm

$$V = i - \frac{2e^t}{-ie^t + e^{2+x}} \tag{4.104}$$

şeklinde olur.

Durum 4

$$a_0 = \frac{a_1 b_{-1} + a_{-1} b_1}{\sqrt{b_{-1}}\sqrt{b_1}},\ b_0 = 2\sqrt{b_{-1}}\sqrt{b_1},$$
$$\alpha = \frac{\beta(a_1 b_{-1} - a_{-1} b_1)}{12 b_{-1} b_1},\ \gamma = \frac{\beta(-a_1 b_{-1} + a_{-1} b_1)}{12 b_{-1} b_1}. \tag{4.105}$$

Bu halde genel çözüm,

$$V = \frac{\frac{e^{tc_3} a_{-1}}{\sqrt{b_{-1}}} + \frac{e^{xc_1 + yc_2 + zc_3} a_1}{\sqrt{b_1}}}{e^{tc_3}\sqrt{b_{-1}} + e^{xc_1 + yc_2 + zc_3}\sqrt{b_1}} \tag{4.106}$$

biçiminde olur. Ayrıca,

$$y = z = c_1 = c_2 = c_3 = c_4 = b_{-1} = a_1 = a_{-1} = 1,\ b_1 = 2 \tag{4.107}$$

olacak şekilde seçilirse, özel çözüm

$$V = \frac{1}{2}\left(\frac{2 + \sqrt{2}e^{2-t+x}}{1 + \sqrt{2}e^{2-t+x}}\right) \tag{4.108}$$

şeklinde olur.

5. TARTIŞMA

Bu çalışmada, daha önce çalışılan iki farklı problem ve KdV benzeri problem EFM ile detaylı olarak irdelenmiştir. Burada, EFM yöntemi uygulanırken istenılen ve istenilmeyen sonuçlar tartışılacaktır. (**??**), (**??**) ve (**??**) denklemlerine EFM uygulandı. Burada, (**??**), (**??**) ve (**??**) denklemlerinin aday çözümleri (**??**) denklemi olarak kabul edildi. (**??**) denklemindeki EFM ile parametreler belirlenirken denklemin pay ve payda kısmında bulunan katsayılarla doğru orantılı olabilir. Bu halde, istenilen çözüm veya yaklaşık çözüm yerine sabit çözüm bulunur. Çoğunlukla, EFM ile bulunan çözümler üzerelnde çalışılan denklemi sağlıyor. EFM üzerine yapılan yorumlar için (Kudryashov ve Loguinova, 2009; Navickas vd., 2010) çalışmalarına bakılabilir.

6. SONUÇLAR

Burada, (**??**) denkleminin EFM ile tam çözümleri (**??**) ve (**??**) denklemleri ile verildi. (**??**) denkleminin EFM ile çözümü (**??**) ve bir özel çözümü ise (**??**) denklemi ile verildi. Son olarak, (**??**) denkleminin EFM ile tam çözümleri sırasıyla (**??**), (**??**), (**??**) ve (**??**) ile ifade edildi. Ayrıca, (**??**) denkleminin EFM ile özel çözümleri sırasıyla (**??**), (**??**), (**??**) ve (**??**) denklemleri ile verildi. Bulunan tüm çözümlerin 2D-3D boyutlu grafikleri ve contour grafikleri görselleri bulgular kısmında verilmiştir.

7. ÖNERİLER

EFM yüksek mertebeli denklemlere uygulandığında cebirsel işlemler oldukça uzun olabilir. EFM dezavantajları olmakla birlikte dalga çözümleri elde etmek için etkili ve güvenilir bir yöntemdir. EFM yöntemi yerine farklı yöntemler de kullanılabilir.

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